

Paper / Subject Code: N10411 / Applied Mathematics-I

Question 1 is compulsory.  
Attempt any 3 from questions 2 to 6.  
A Scientific Calculator is allowed to use.

Applied Mathematics-I  
Revised Course (NEP-2020)  
(June 2025)

Marks-60  
Duration-02 Hours

1. Attempt any Five questions. (Compulsory Problem)

- (a) Prove that  $\sqrt{1 + \operatorname{cosec}(\theta/2)} = (1 - e^{i\theta})^{-1/2} + (1 - e^{-i\theta})^{-1/2}$ . 3
- (b) Find  $n^{\text{th}}$  derivative of  $\frac{2}{(x-1)(x-2)(x-3)}$ . 3
- (c) If  $5 \sinh x - \cosh x = 5$ , find  $\tanh x$ . 3
- (d) Show that the matrix  $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$  is unitary and hence find  $A^{-1}$ . 3
- (e) If  $u = 2(ax + by)^2 - k(x^2 + y^2)$  and  $a^2 + b^2 = k$ , find  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ . 3
- (f) Using Newton-Raphson Method, find the real root of  $x^3 - 2x - 5 = 0$  correct to three decimal places. 3
2. (a) If  $\tan[\log(x + iy)] = a + ib$ , prove that  $\tan[\log(x^2 + y^2)] = \frac{2a}{1 - a^2 - b^2}$  when  $a^2 + b^2 \neq 1$ . 4
- (b) Examine the function  $f(x, y) = y^2 + 4xy + 3x^2 + x^3$  for extreme values. 5
- (c) If  $u = f(e^{x-y}, e^{y-z}, e^{z-x})$ , then prove that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ . 6
3. (a) Reduce the following matrix to normal form and find the rank of matrix  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$ . 4
- (b) Solve  $x^6 + 1 = 0$ . 5
- (c) Apply Gauss-Seidel iteration method to solve the following linear equations up to the three iterations. 6  
 $20x + y - 2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25.$
4. (a) Expand  $\sin^5 \theta \cos^3 \theta$  in a series of sines of multiples of  $\theta$ . 4
- (b) Investigate for what values of  $\lambda$  and  $\mu$  the equations  $x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$  5  
Will have (i) No Solution (ii) Unique Solution (iii) Infinitely Many Solutions.
- (c) If  $y = a \cos(\log x) + b \sin(\log x)$ , prove that 6  
 $x^2 y_{n+2} + (2n + 1)x y_{n+1} + (n^2 + 1)y_n = 0$
5. (a) If  $\cos \alpha + \cos \beta + \cos \gamma = 0, \sin \alpha + \sin \beta + \sin \gamma = 0$ , prove that 4  
i)  $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$   
ii)  $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$
- (b) If  $\alpha + i\beta = \tanh\left(x + \frac{in}{4}\right)$ , prove that  $\alpha^2 + \beta^2 = 1$ . 5
- (c) If  $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x+y}}\right)$ , prove that  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4 \cos^3 u}$ . 6
6. (a) Find all the roots of  $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{3/4}$ , hence find the continued product of the roots. 4
- (b) Show that the minimum value of  $u = xy + a^3\left(\frac{1}{x} + \frac{1}{y}\right)$  is  $3a^2$ . 5
- (c) Find non-singular matrices P & Q such that PAQ is in normal form & hence find rank of the matrix A and obtain  $A^{-1}$ . 6

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$